Modelling crack initiation in low cycle fatigue: a review

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Abstract. The typical fatigue life of a component is mainly divided into two phases: crack initiation, and crack propagation. This study is concerned with the crack initiation life as some designers regard the crack initiation as the end of the design life of the component. Crack initiation is caused by the formation of persistent slip bands that interact with the matrix leading to embryonic crack formation. There are several studies defining the point of crack initiation, and this article addresses some of these definitions. The main aim of this study is to review the different modelling methodologies for crack initiation under low cycle fatigue. These models are divided into three main types: microscopic models, damage parameters, and probabilistic models. There is no preferred methodology among the ones discussed. The choice of which model to use depends on the type of loading, the material in use, and the required level of detail. This study is intended as a reference for using one of these models or introducing modifications to enhance them.

Keywords: low cycle fatigue, crack initiation, damage parameters, strain energy, SWT

1 Introduction

The study of fatigue loading for structural components is essential for several applications. In fact, fatigue damage is regarded as the most common damage mode for these components. The applications include aerospace, production machinery, and energy producing power plants. Fatigue loading is divided into two main types: Low cycle fatigue (LCF), and High cycle fatigue (HCF). This study is focused more on LCF loading, but the principles can be applied to both types.

Under LCF loading, the typical fatigue life of a component is divided into three phases [1, 2]:

(1) Crack initiation in which micro-cracks appear on the specimen surface. However, these cracks are retarded and none of them is dominant. The most dominant phases is lasting for 40-90% of the total fatigue life.

(2) Stable crack growth in which damage is localized by one dominant crack starting to propagate.

(3) Unstable crack growth, which is a rapid phase leading to the failure of the component.

This process is demonstrated in **Fig. 1**. It is worth noting that sometimes crack initiation stops at barriers such as grain boundaries and then the crack remains in this level and never reaches the critical size leading to the second or third phases.

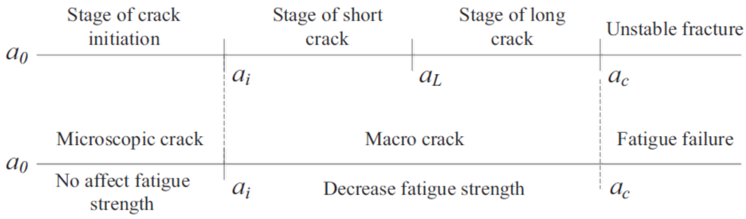

Fig. 1. Schematic diagram showing the fatigue failure process [3]

Fatigue crack initiation takes place when a micro-crack propagates to an engineering measurement dimension [4]. For example, Bhattacharya [5] defined a crack length of 0.076 mm as critical crack initiation size. Murtaza and Akid [6] defined it as 0.12 mm. Rodopoulos [7] made a generalized assumption that crack initiation started when the crack length became several times the size of the microstructure.

Under this framework, the fatigue life of a smooth component can be calculated as follows:

$$N_f = N_i + N_p \quad (1)$$

Some studies provided a relation between the fatigue crack initiation point and the cyclic softening behaviour of the material. These studies have been applied whenever the macroscopic detection of crack initiation is not possible. Zhang et al. [8] showed that crack initiation occurred at approximately 80% of the fatigue life under uniaxial loading based on Damage Mechanics theory. Another approach regards that crack initiation takes place at 10% drop in the stress amplitude [9, 10].

In this study, the mechanism of crack initiation is explained and then linked to the different macroscopic and microscopic modelling techniques.

2 Mechanism of crack initiation

Fatigue crack initiation is a random and complex process, which depends on the plastic strain amplitude, and the environment. The detailed process is summarized by Leon-Czares [11] as follows:

1) Redundant dislocation density is generated due to fatigue hardening or softening to form a stabilized dislocation population.

2) Constrained dislocation substructure is formed leading to the localization of slip into “Persistent Slip Bands (PSB)”.

3) The dislocation substructure interacts with a free surface producing extrusions and associated intrusions on it. This gives rise to the precipitation of free zones near the surface.
4) Stress incompatibility is caused by this intrusion leading in the end to the production of embryonic cracks.

This process can be confirmed by microstructural investigations. **Fig. 2** shows the slip band formation, and the initiation of the crack at the interface between the slip band and matrix.

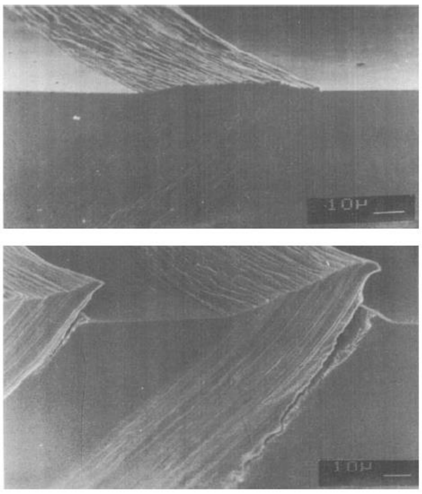

Fig. 2. Slip band formation as a result of fatigue loading followed by the formation of crack[12]

3 Crack initiation modelling

3.1 Microscopic modelling

In order to represent the microstructure of the material, several techniques could be used. The immediate mapping of a microscopic image in a geometric finite element mesh is one of the most straightforward techniques, which leads to the matching between simulation and experimental results [13, 14]. The disadvantage of this method is that the simulation is strictly related to the mapped area. This is why other techniques have been developed to extend the simulation to the whole specimen. One technique is the statistical based synthetic microstructure using statistical information such as phase fraction and grain size distribution obtained from microscopic images. This information can be processed into a representative volume elements (RVE) model [15-18]. In the end, the mean material properties are given for the continuum.

A study by Tanaka and Mura [19] was one of the pioneering studies to present a model relating the fatigue crack initiation with the accumulation of dislocation dipoles that are generated by irreversible slip bands in one grain. In this model, the strain energy of dislocations is accumulated by the same amount during forward or reverse loading.

A crack initiates when the accumulated energy reached fracture energy. The equation used to estimate crack initiation life is given by:

$$N_c = \frac{8GW_s}{\pi(1-\nu)d(\Delta\tau-2k)^2} \quad (2)$$

Where G is the shear modulus, ν is the Poisson's ratio, W_s is the specific fracture energy per unit area, and $\Delta\tau$ is the resolved shear stress range on the slip band. Also, k is the frictional stress of dislocation on the primary slip plane, and d is the length of the slip band.This model was used by Zimmermann and Rie [20], and by Huang [21] for strain controlled fatigue loading on ferrous and non-ferrous metals. It was also used by Hoshide and Kusuura [22] for multiaxial fatigue loading.

Crystal plasticity (CP) models have also been used in several studies to calculate the dislocation reactions on slip systems. It was initially developed for single crystals [23], but was then further enhanced to include polycrystalline features [24-27]. These features include dislocation density [28], grain boundary mechanisms [29-32], and damage initiation [33].

Gu et al. [34] used the crystal plasticity model to predict the crack initiation around the inclusions. In this model, the local accumulated dislocation p_{acc} was calculated using the following equation:

$$p_{acc} = \int_0^t \sqrt{\frac{2}{3} L_p : L_p}\, dt \quad (3)$$

Where L_p is the plastic velocity gradient that is calculated based on the plastic deformation gradient. This represents the shear stress, the slip systems, and the dislocation densities. This model allowed for the detection of the different crack nucleation sites in the microstructure.

3.2 Damage parameters

Thermodynamic damage parameter. A study by Zhang et al. [35] was able to successfully simulate the fatigue crack initiation at the inclusion-matrix interface. This work was based on the thermodynamic theory of damage mechanics, in which the fatigue damage is regarded as an irreversible thermodynamic process [36]. This process dissipates the internal energy in the form of heat. A damage characterization parameter Y is introduced, which is related to the material behaviour as:

$$Y_{max} = \frac{E}{2} {\varepsilon_{max}}^2 \quad (4)$$

The damage characterization parameter can then be related to the time dependent damage D, representing the free energy dissipation caused by the internal damage, by the following equation:

$$\frac{dD}{dN} = aK{Y_{max}}^{m/2} \quad (5)$$

The FE simulation of this study was performed on ANSYS using the Debonding submodule in the Contact Analysis module. This module is commonly used for cracking and separation of the material.

Strain energy damage parameter. The energy-based damage parameters usually possess specific physical significance, and more importantly they can be used on high-cycle and low-cycle fatigue because they involve both stress and strain components [37].

A damage parameter was used by Yeunyong [38] merged with the Chaboche viscoplasticity model to detect the fatigue crack initiation. This damage parameter is based on the hysteresis energy criterion accounting for the accumulation of stabilized hysteresis strain energy. This damage criterion was used in several other studies [39-41]. The accumulated inelastic strain energy per cycle ΔW with the number of cycles N is formulated by the following equation:

$$N = A\Delta W^B \tag{6}$$

In which A and B are material parameters. The progression of the damage parameter with the number of cycles is given by:

$$\frac{dD}{dN} = \frac{1}{A\Delta W^B} \tag{7}$$

This damage parameter affects the stress tensor (σ) causing the material softening progressively damaged until failure when the value of D reaches 1. The effect is given as:

$$\sigma = (1 - D)\bar{\sigma} \tag{8}$$

Critical plane (CP) approach. Strain energy is scalar quantity, and this is why most energy-based models cannot solely describe crack initiation and propagation on a specific plane for most metallic materials [42, 43]. This raises the need for the critical plane (CP) approach to detect the favourable plane for the crack to initiate. Smith-Watson-Topper [44] proposed a damage parameter to describe the crack initiation in a specific plane. This parameter depends on the maximum normal stress, and the true normal strain range on a specific plane as given by:

$$P_{SWT} = \sigma_{t,max} \frac{\Delta\varepsilon_{t,T}}{2} \tag{9}$$

Where $\sigma_{t,max} and\ \Delta\varepsilon_t$ are the maximum normal stress and the true normal strain range respectively.

The SWT parameter was proved to be incapable of detecting damage in many materials, especially the ones dominated by shear crack initiation [45, 46].
Another damage parameter was proposed by Findley [47] taking into account the effect of the normal stress on the maximum shear stress plane. The introduced critical plane model predicts the fatigue crack plane with orientation θ having maximum Findley Damage parameter. This is expressed by:

$$f(\theta) = \frac{\Delta\tau}{2} + k\sigma_n \tag{10}$$

Where $\Delta\tau/2$ is the shear stress amplitude on a plane with orientation θ, σ_n is the maximum normal stress on that plane, and k is a material parameter.

Several versions of this damage parameter have been developed. Brown and Miller considered both the shear and normal strain on the plane of maximum shear stress. This is based on the assumption that cyclic shear strain will help in the crack nucleation, and the normal strain will help in its growth. A simplified formula was proposed by Kandil, Brown, and Miller[48] given by:

$$\frac{\Delta\hat{\gamma}}{2} = \frac{\Delta\gamma_{max}}{2} + S\Delta\varepsilon_n \tag{11}$$

where $\Delta\gamma_{max}$ is the maximum shear strain range, and $\Delta\varepsilon_n$ is the maximum normal strain range on the plane with the maximum shear strain range. *S* is a material parameter representing the influence of the normal strain on crack growth.

The different critical plane methods have the benefit of being used to model multi-axial fatigue loading. This is why these methods have been used in various studies [49, 50].

3.3 Probabilistic modelling

As discussed previously, crack initiation process is a random process that depends on several uncertainties such as geometrical features, and microstructures (presence of inclusions, or defects). This is why sometimes deterministic models become inaccurate especially on the attempt of generalizing these models.

Some studies have been calling for some statistical arguments in order to apply the different models for design purposes. From here, probabilistic models were established in order to carry out reliability analysis. A study by Castillo and Fernandez-Canteli proposed a probabilistic model based on Weibull or Gumbel distributions[51]. This model starts with one damage parameter such as the life to initiate a macrocrack N_f and expressed by:

$$\psi = q(N_f) \tag{12}$$

ψ here represents the fatigue damage parameter, and *q* represents a decreasing function of total life in terms of reversals to macro-crack initiation. This damage parameter is then coupled with a probabilistic Weibull regression model in the following compact form:

$$f(\psi)f(N_f) = \nu \tag{13}$$

where ν here is a regression parameter and *f* is logarithmic function with the following form:

$$f(x) = log(\mathrm{x}) - \theta_\mathrm{i} = log\left(\frac{\mathrm{x}}{\mathrm{e}^{\theta_\mathrm{I}}}\right) \tag{14}$$

In this equation, i = 1,2, $\theta_1 = \psi_0$, and $\theta_2 = N_0$. These parameters can be extracted from experimental data. Then, a set of values ν_k (where k = 1,2,… till the number of experiments) are fitted with a three-parameter Weibull distribution(**Fig. 3**):

$$N_k \sim W(\lambda, \delta, \beta) \tag{15}$$

The parameters of this distribution are: λ defining the position of the corresponding zero percentile hyperbola, δ defining the scale factor, and β representing the Weibull shape parameter of the cumulative distribution variable.

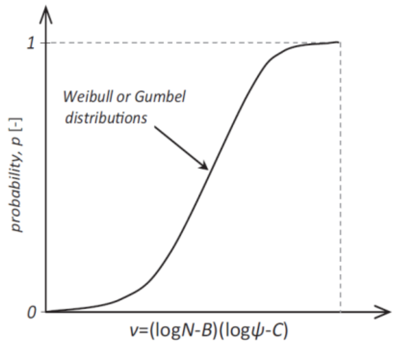

Fig. 3. Schematic diagram of the probability of regression parameter[52]

The advantage of this probabilistic model is that it can be coupled with various damage parameter models turning it from deterministic to probabilistic. It has successfully been used in combination with Chaboche deterministic method [53], multiaxial fatigue virtual strain energy model [54], and Miner's linear damage summation [55]. The model was also used to estimate fatigue life on structural components.

Other probabilistic approaches have been studied such as the model by Zhu et al. [56] using Bayes theorem, and Koutiri et al. [57] using the weakest link concept.

4 Conclusion

In this review, several modelling methodologies have been discussed to predict the low cycle fatigue crack initiation lifetime. These methodologies can be microscopic, macroscopic, or statistics based. Some models were able to cover different applications: low cycle fatigue, high cycle fatigue, uniaxial loading, multiaxial loading, etc, while other models had limitations. Also, not every model can be applied on every type of material, so care must be made when choosing the suitable model. In the end, there is no preferred model out of the ones in discussion. It is also important to mention that there is still room for improvement in these models and correlating them with the physics of crack initiation. As discussed, crack initiation is a complex and random process with no agreed upon definition.

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